MADALGO
CENTER FOR MASSIVE DATA ALGORITHMICS


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Accelerating Dynamic Programming

What
Dynamic Programming is a fundamental problem-solving technique that has been widely used for efficiently solving a broad range of search and optimization problems.

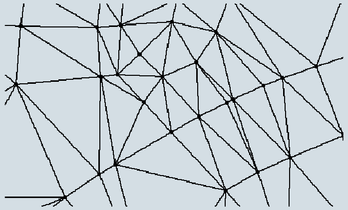
Problem
Devise a unifying framework for speeding up dynamic programming algorithms.

Solution
A toolkit of acceleration tricks such as text-compression, total monotonicity, partial DP tables, utilizing sparsity, fractional subproblems, Four-Russians speedup, fast min-plus matrix multiplication, and bounded treewidth algorithms.

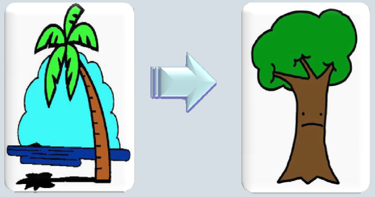
Usage
Our speedups apply to many fundamental computational problems such as shortest paths, matrix multiplication, edit distance, HMM decoding and training, array searching, tree searching, and problems on graphs of bounded treewidth.

Acceleration via Text-Compression
We use compression to identify repeats and then exploit them to accelerate dynamic programming. We show how *Straight-line programs* can be used to capture most compression schemes and to speed-up problems such as *string edit-distance* and *decoding and training Hidden Markov Models.*

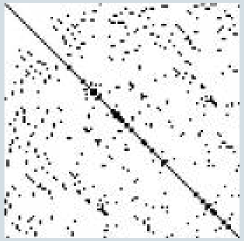
Total-Monotonicity & The Knuth-Yao Quadrangle-Inequality
We show that planar graph problems such as *shortest paths, replacement paths, bipartite perfect matching, and feasible flow,* can be accelerated by utilizing a total-monotonicity property of the paths.



Partial Dynamic Programming Tables
We show how to compute *tree edit-distance* by filling only a subset of the dynamic programming table (i.e. only computing relevant subproblems).



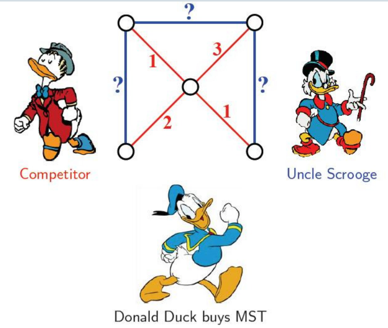
Utilizing Sparsity
We can use the sparsity inherent to some problems such as *tree LCS.* We find for each point of the sparse problem a geometric region of the DP table in which that point can influence the values of other points.



Fractional Subproblems
We suggest a method of reusing only parts of a subproblem's structure and extending this part into many more larger ones. We show that this can be used for finding the optimal tree-searching strategy (i.e. binary searching a tree) and for file-system synchronization.

Generalized Four-Russians Method
We suggest sophisticated identifiers to be used as pointers for the Four-Russians table-lookup speedup. This has applications to the *Range Minimum Query* problem and to generalizations of *Cartesian trees.*

Bounded Treewidth Graphs
We present the first dynamic programming solution for an NP-hard two-player game on bounded treewidth graphs. Namely, the *Stackelberg Minimum Spanning Tree Game.*



Min-Plus Matrix Multiplication
Many dynamic programming problems reduce to computing the min-plus product of two matrices. We explore various problems that admit restricted matrices whose min-plus product can be computed efficiently.

$$A = B \oplus C$$

$$A_{i,j} = \min\{B_{i,k} + C_{k,j}\}$$

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